CONTROL and COMPARISON of A TIME-DELAYED SYSTEM USING A PD, 2DOFPD, and FOPD CONTROLLER

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ABSTRACT

Time-delayed systems are prevalent in industrial processes and are commonly encountered in chemical and fluid systems, manufacturing plants, communication networks, and defense industry operations. This study employs PD, fractional order PD (FOPD), and two degrees of freedom PD (2DOFPD) controllers to manage an example of a time-delayed system. In addition to analyzing the responses to unit step input, the system's resilience to introduced disturbances was also tested and the results were examined. In addition, the controllers' efficacy against errors was evaluated using performance indices such as Integral Square Error (ISE), Integral of Absolute Value of Error (IAE), Integral of Time-weighted Absolute Error (ITAE), and Integral of Time-weighted Square Error (ITSE). Additionally, these controllers were recalibrated through precise adjustments, and a subsequent performance comparison was conducted. The simulation of the system was fully conducted in the Matlab/Simulink environment, and the results are presented for further discussion.

Keywords: time delay system, fractional order PD, two degrees of freedom PD, integral of time absolute error, control systems.

Introduction

Time-delay systems are commonly encountered in industrial processes, such as fluid-gas flow, and in communication systems where time delays occur. The natural time delay in the system causes problems in process control. To overcome this issue, controllers must be adjusted to tolerate this time delay. This situation leads to different controllers achieving different outcomes. Determining which controller to use in a given situation, or which type will be sufficient, presents a significant challenge (Åström & Hägglund, 2001; Dorf & Bishop, 2020). Therefore, alongside the most fundamental industrial controller, the PID controller, others such as 2DOFPID, KDPID, and their versions with added filters can be used for comparison purposes (Dong et al., 2024; Xia et al., 2023; Sundaravadivu, 2015). Additionally, controllers derived from artificial intelligence, such as fuzzy logic controllers (Manuel et al., 2023) and genetic algorithms (Gürler et al., 2023), can also be used (Chen et al., 2023). Moreover, hybrid-structured controllers such as fuzzy-tuned PID are also available (Liu et al., 2023).

Determination of controller parameters and evaluation of their performance for comparison require certain parameters. This helps to identify the aspects in which one controller performs better than others. For this purpose, error performance indices such as integral square error (ISE), integral of the absolute value of error (IAE), integral of time-weighted absolute error (ITAE), and integral of time-weighted square error (ITSE) values are commonly used (Mengi et al., 2022). In addition, parameters such as steady-state error value, maximum overshoot, and settling time are also considered in the design (Ayas & Sahin, 2021).

In this study, a time-delayed sample system was controlled using PD, 2DOFPD, and FOPD controllers, and efforts were made to highlight the differences between these controllers. The results are presented both graphically and numerically in tables.

Materials and Methods

This study compared the performance of three different types of controllers to identify their distinct characteristics. The controllers examined were PD, 2DOFPD, and FOPD types. These were tested in a simulated industrial process that mirrored the typical time delays found in real-world industrial environments.

The experimental setup was designed to independently evaluate the performance of each controller. Key aspects such as response time, stability, and adaptability to time delays of the controllers were scrutinized. For the evaluation process, error performance indices such as ISE, IAE, ITAE, and ITSE were used. Additional parameters, including steady-state error, maximum overshoot, and settling time, were also considered.

This approach ensures a comprehensive analysis of each controller, allowing for an in-depth understanding of their operational capabilities and limitations within a time-delayed process environment.

Time-Delayed Systems

Time-delayed systems are complex dynamic systems that are commonly encountered in industrial and engineering applications. These systems are usually expressed mathematically, as shown in Equation 1. The key characteristic of this equation is that it reflects how the system's past states affect its current state, which is identified as time delay. This time delay may result from various factors, such as the speed of the control cycle or the communication network in the system.

Analyzing these systems is more complex than that of traditional control systems and can significantly impact the stability and performance of the systems. The dynamics of time-delayed systems are often observed in fluid and gas flows, communication networks, and even economic models. Accurate modeling and analysis of these systems are crucial for the development of effective control strategies.

In this study, the mathematical modeling of time-delayed systems is based on the approach proposed by Luyben (Luyben, 2003). Luyben's model, commonly used in industrial process control, reflects the fundamental characteristics and control challenges of these systems. This modeling serves as a cornerstone for testing and evaluating the performance of the controllers used in this study.

$$G(s) = \frac{K(-\tau_a s + 1)e^{-\theta s}}{s(\tau s + 1)} \tag{1}$$

PID Controller

PID controllers are widely favored because of their simplicity, affordability, quick response, and user-friendly nature (Güven & Duman, 2017). Commonly, their parameters K_P, K_I, and K_D are tuned using the Ziegler–Nichols technique. Further adjustments are made through various methods such as optimization algorithms to meet specific requirements. The formula for the PID controller is shown in Equation (2), and its design is depicted in Figure 1.

$$c(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_i}{s} + K_d s \tag{2}$$

K_P, K_I, and K_D are PID controller gains. Here, *U(s)* is the control signal and *E(s)* is the error signal (Mengi, 2020).

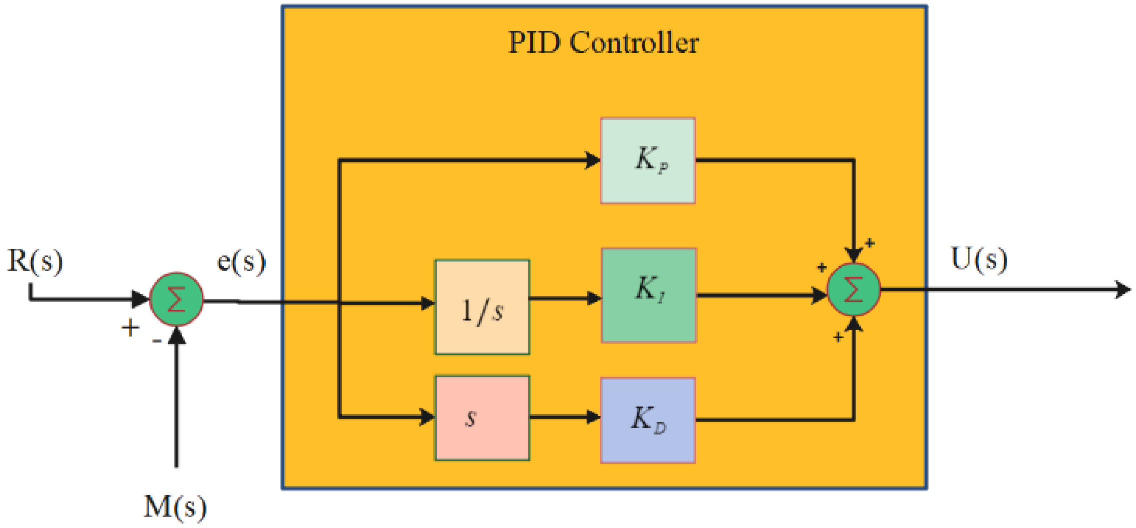


Figure 1. ***PID controller***

2DOFPID Controller

2DOFPID controllers are more sophisticated and precise variants of PID controllers. They are engineered to uphold superior performance in demanding and varying system conditions. Uniquely, they afford two degrees of freedom for both error and its derivative (Güven, 2023). This added attribute allows the 2DOFPID controller to deliver enhanced control and adaptability compared with a conventional PID controller in numerous situations. Their mathematical representations are shown in Equations (3) and (4).

$$u = K_P[(br - y) + \frac{1}{T_i s}(r - y) + \frac{T_D s}{\frac{T_d}{N}s + 1}(cr - y)] \tag{3}$$

$$u = K_P(br - y) + \frac{K_i}{s}(r - y) + \frac{K_d s}{T_f s + 1}(cr - y) \tag{4}$$

Here c is set point weighting (derivative), b is set point weighting (proportional), T_f is derivative filter time (s), T_i is integrator time (s), T_d is derivative time (s) and N is filter coefficient (Thakallapelli et al., 2016). This structure is shown in Figure 2.

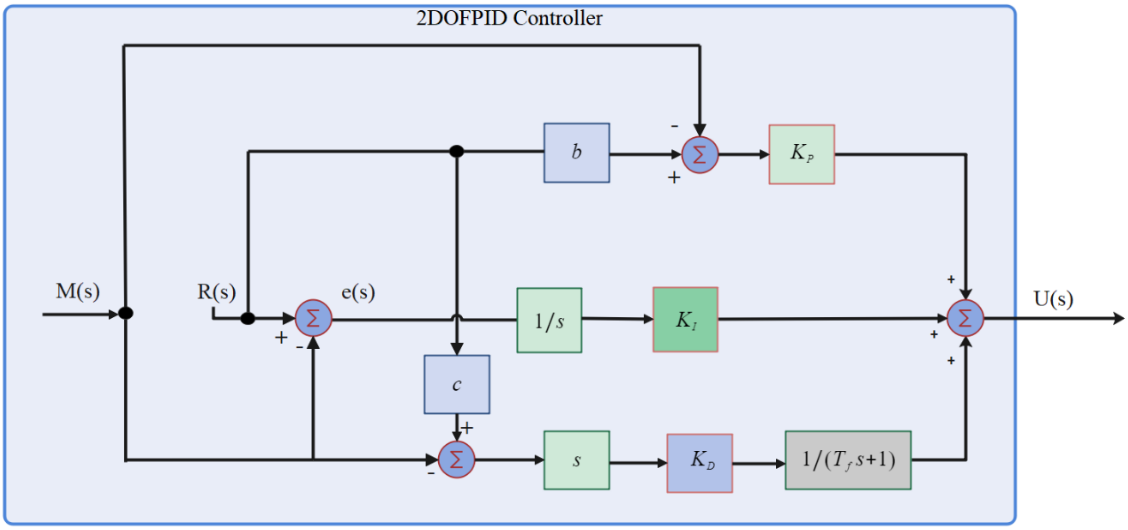


Figure 2. ***2DOFPID controller***

Fractional-Order PID Controller

A fractional-order PID (FOPID) controller is an enhanced form of the conventional PID controller, characterized by a fractional order of the integral operator (λ) and a fractional order of the derivative operator (μ) (Ahmed et al., 2023). The transfer function for the FOPID controller is provided as follows.

$$G_{CONTROLLER} = K_P + \frac{K_i}{s^{\lambda}} + K_D s^{\mu} \tag{5}$$

The controller is designed with five adjustable parameters: K_P, K_i, λ, K_D, μ. These extra parameters offer greater control and enhanced capability to counter disturbances, leading to improved system dynamics over traditional PID controllers. Figure 3 illustrates the FOPID controller's block diagram.

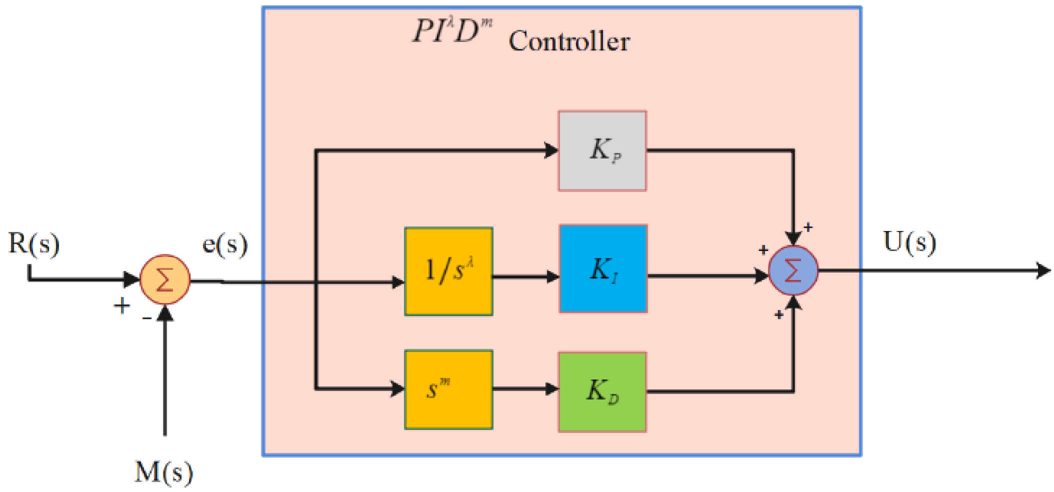


Figure 3. $PI^{l}D^{m}$ controller

Performance Indices

To evaluate the effectiveness of PID controllers, various performance indices were employed. Notably, IAE, ITAE, ISE, and ITSE are among the most commonly used. These indicators are assessed on the basis of the evolution and cumulative value of the error over a given period. In this study, the ITAE index was selected for performance measurement. The formulas for these indices are outlined in Equations (6) to (9) (Mengi, 2022).

$$\mathrm{ISE} = \int_0^{\infty} e^2(t)\,dt \tag{6}$$

$$\mathrm{IAE} = \int_0^{\infty} |e(t)|\,dt \tag{7}$$

$$\mathrm{ITSE} = \int_0^{\infty} te^2(t)\,dt \tag{8}$$

$$\mathrm{ITAE} = \int_0^{\infty} t|e(t)|\,dt \tag{9}$$

To calculate the ITAE value, a specialized computational module was developed using the Matlab/Simulink platform.

Findings and Discussion

The simulated system is illustrated in Figure 4. The transfer function of the time-delayed sample system is presented in Equation 10 as follows:

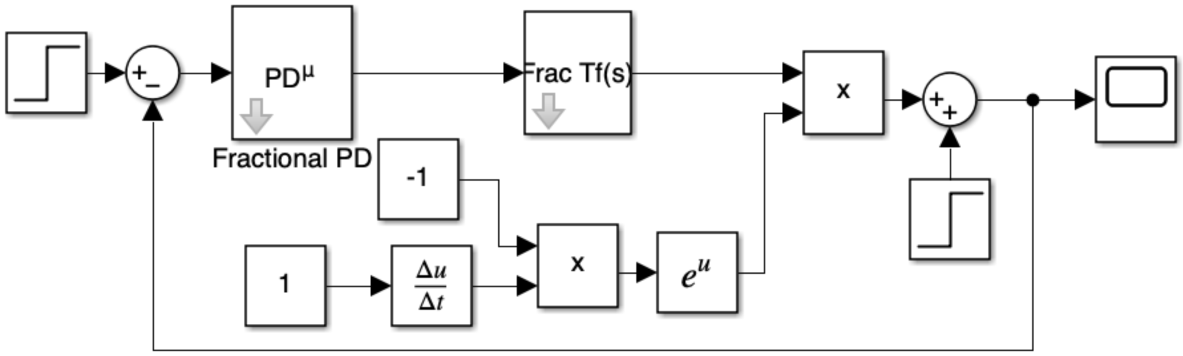


Figure 4. Simulated system

$$G(s) = \frac{1}{s^{1.9} + s} e^{-s} \tag{10}$$

For this system, PD-type controllers have been recommended in the literature (Özyetkin et al., 2018). The performance of the 2DOFPD and KDPD controllers was compared with that of standard PD-type controllers. Subsequently, efforts have been made to obtain more effective results using fine-tuning features in the Matlab/Simulink environment.

Study 1:

Figure 5 displays the outputs obtained using the controller parameters referenced in the literature. In this case, a disturbance of 0.2 units was introduced to the system at t=15s to also test the effectiveness of the controllers. A unit step function served as the reference input and was applied at t=1s.

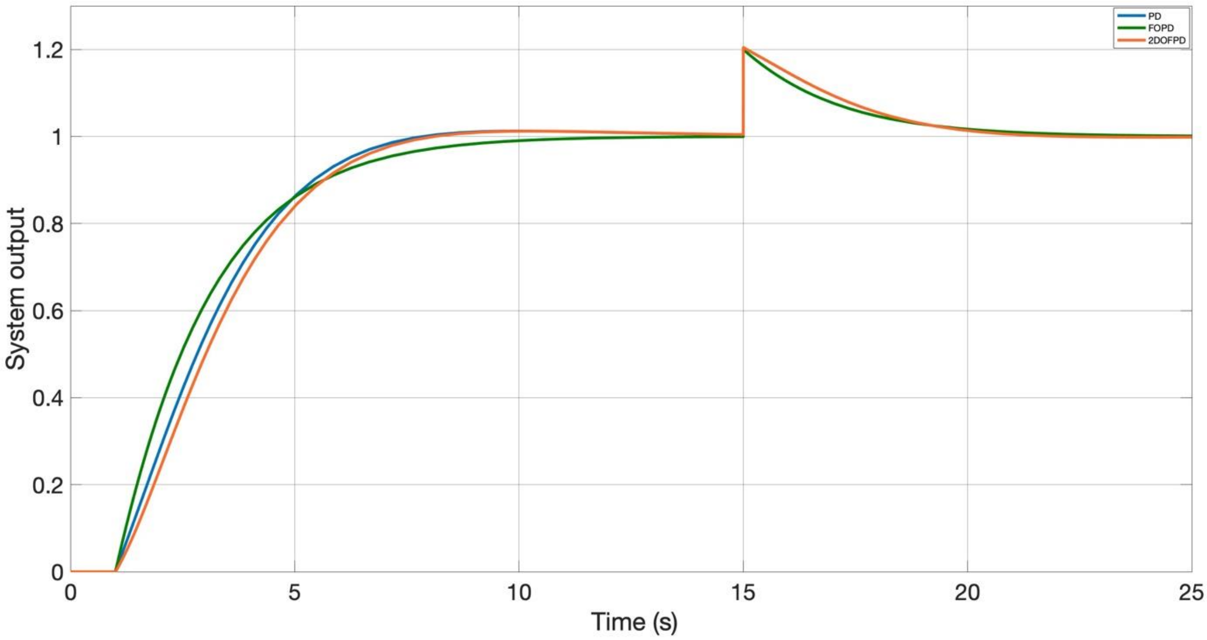
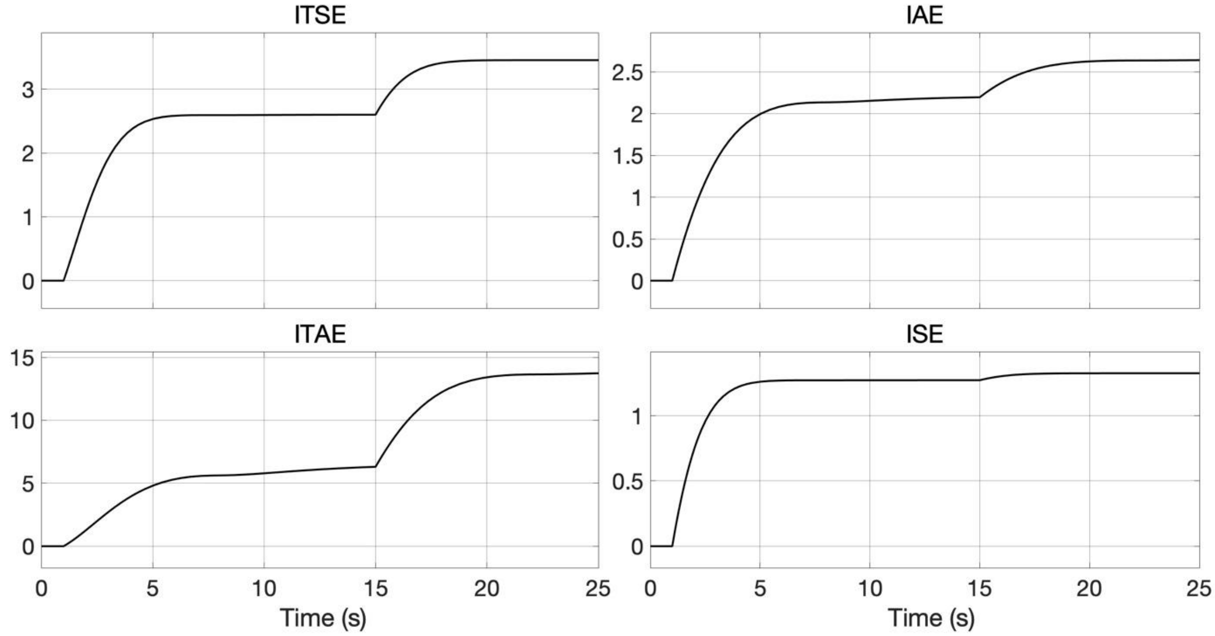
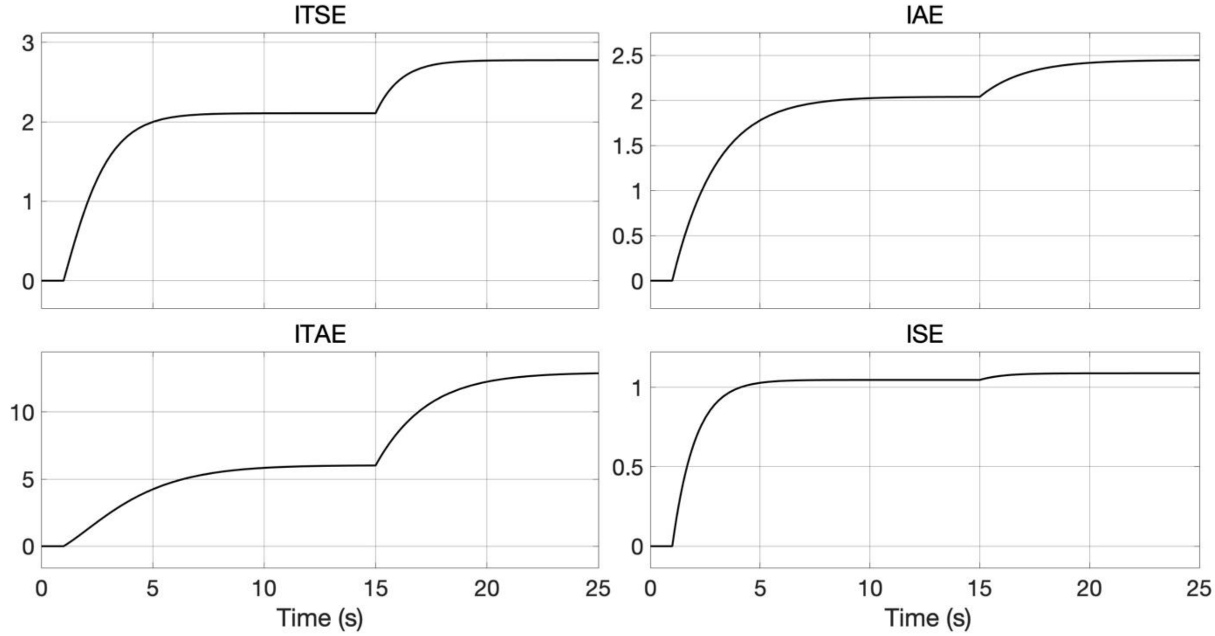


Figure 5. Variation of system output

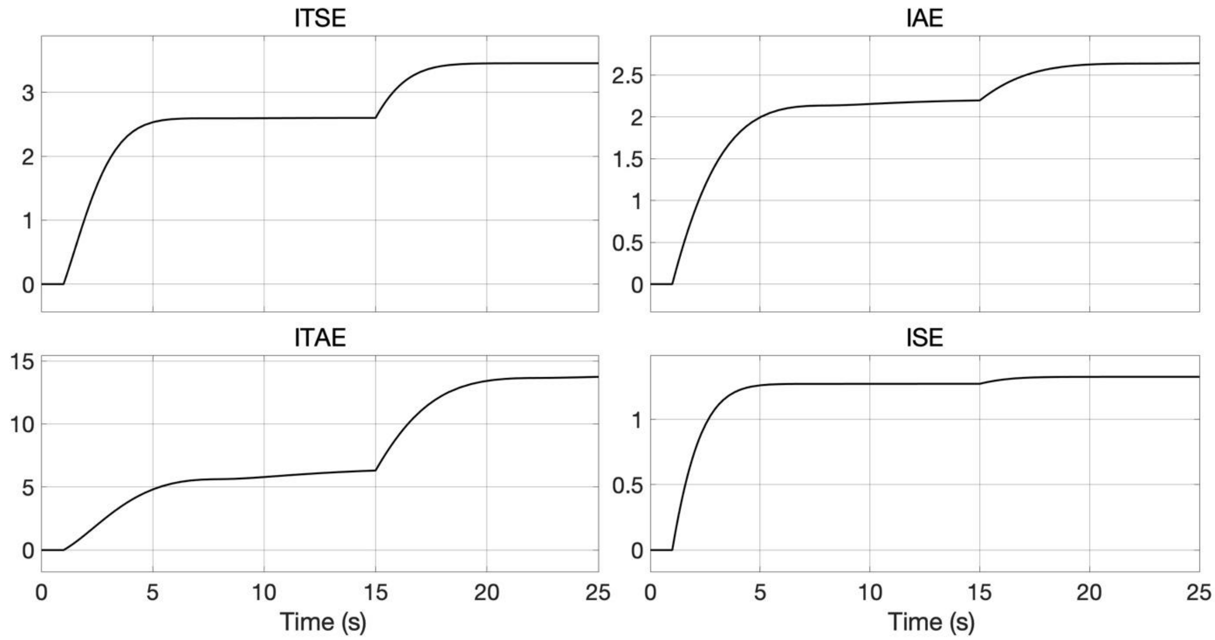
Figure 6 (a) shows the variation of error indices for the PD controller, Figure 6 (b) for the KDPD controller, and Figure 6 (c) for the 2DOFPD controller.



(a) PD



(b) KDPD



(c) 2DOFPD

Figure 6. Variation of error Indices for the PD controller

Table 1 presents a comprehensive view of the controller parameters and the error index values.

Table 1. Controller parameters and performance index values

Controller	K_P	K_I	K_D	λ	μ	b	c	ISE	IAE	ITSE	ITAE
PD	0.4906	-	0.4532	-	-	-	-	1.32	2.63	3.45	13.73
2DOFPD	0.4906	-	0.4532	-	-	1	0.6	1.32	2.63	3.45	13.73
KDPD	0.4906	-	0.4532	-	0.9	-	-	1.08	2.44	2.77	12.89

If observed closely, it can be seen that except for KDPD, the other two controllers yielded almost identical results. In terms of the same parameters, the KDPD controller showed the best performance. As indicated in Table 2, when comparing these three controllers, it is evident that the error values are significantly low. However, the settling times for KDPD are lengthy, at 14 s. In contrast, the others have settled at the unit step value with a very small error margin.

Table 2. Controller response

Controller	M_P	t_s (s)	e_{ss}
PD	0.1	-	0.042
2DOFPD	0.1	-	0.045
KDPD	0	14	0

Study 2:

Figure 7 shows the outputs obtained after the controllers in Matlab/Simulink are fine-tuned. As in the previous study, a disturbance of 0.2 units is added to the system at t=15s. A unit step is used as the reference and is applied at t=1s.

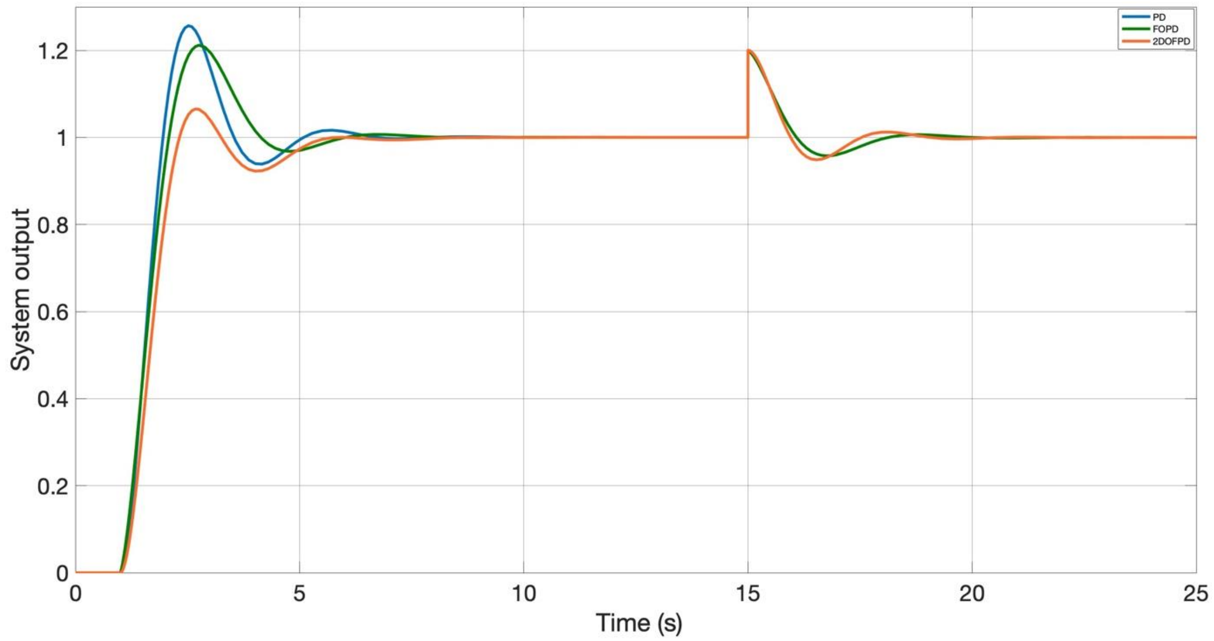
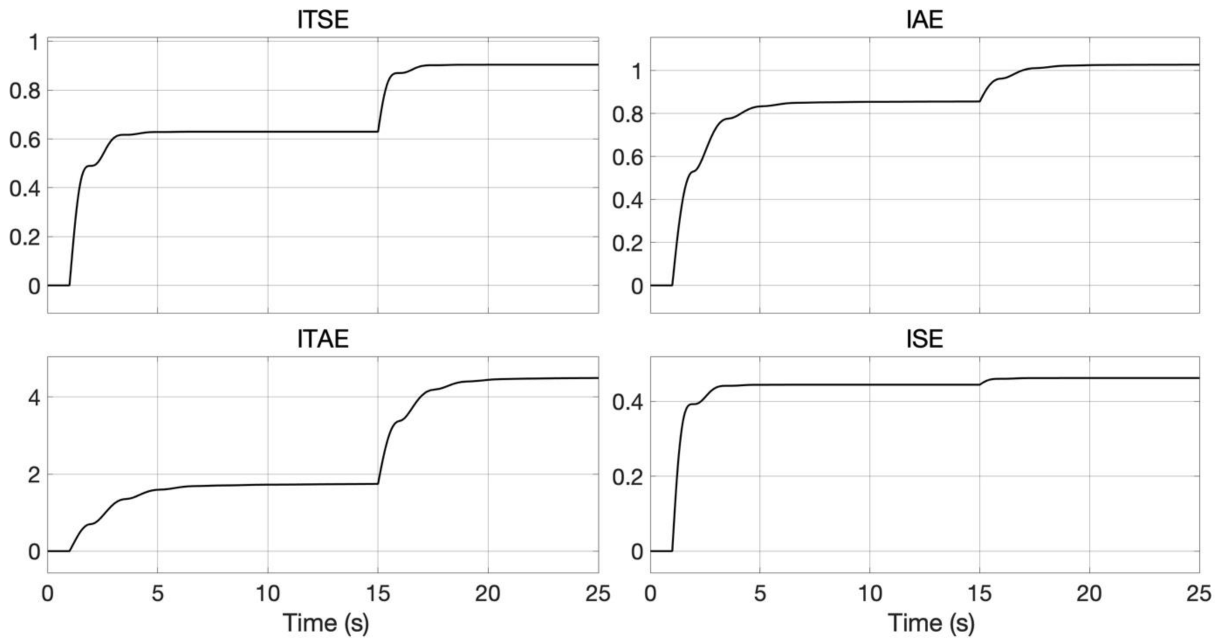
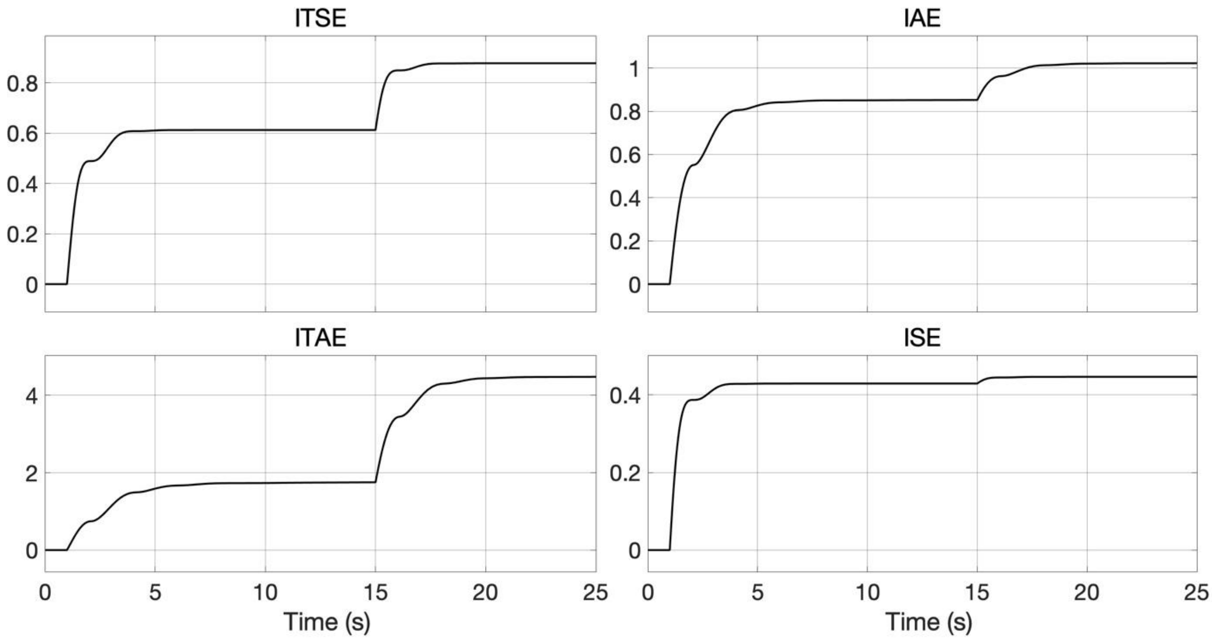


Figure 7. Change in demand output using fine-tuned controllers

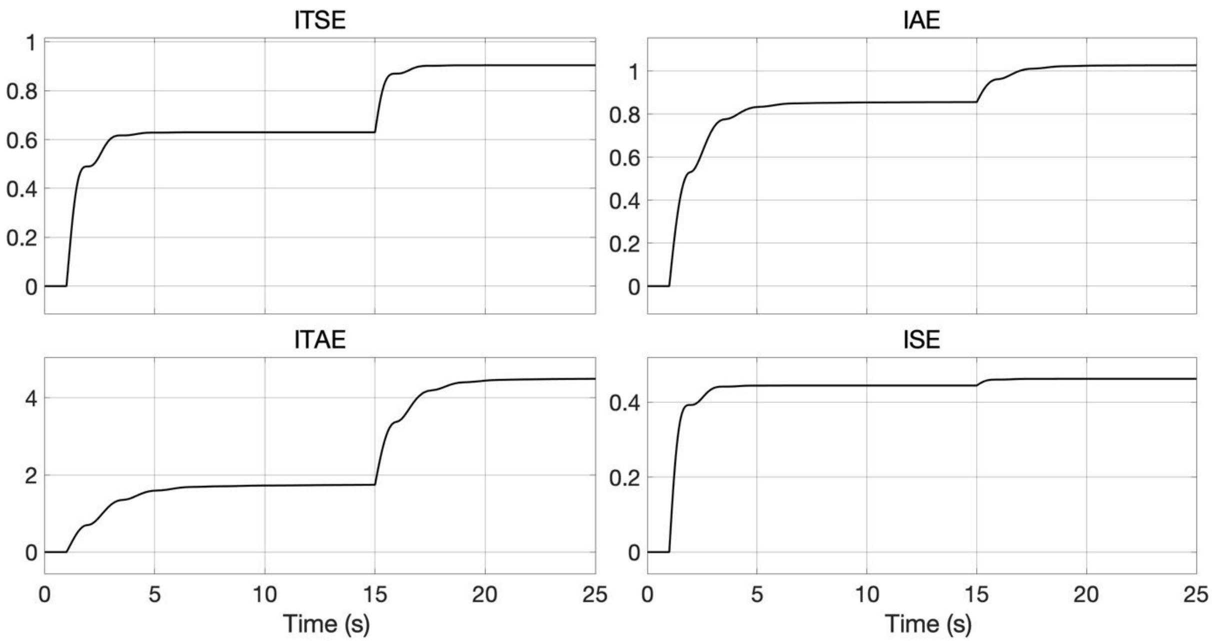
In Figure 8a, the changes in error indices of the PD controller, in Figure 8b, the KDPD controller, and in Figure 8c, the 2DOFPD controller are shown.



(a) PD



(b) KDPD



(c) 2DOFPD

Figure 8. Variation of error indices for the PD controller

Table 3 presents a comprehensive view of the controller parameters and their corresponding error index values.

Table 3. Controller parameters and performance index values

Controller	K_P	K_I	K_D	λ	μ	b	c	ISE	IAE	ITSE	ITAE
PD	3.133	-	0.415	-	-	-	-	0.46	1.027	0.9	4.49
2DOFPD	3.133	-	0.415	-	-	1	0.006	0.46	1.027	0.9	4.49
KDPD	3.133	-	0.415	-	0.85	-	-	0.446	1.022	0.877	4.478

In the analysis conducted after fine-tuning, it was observed that all values closely approached each other. This situation has made interpreting the error indices challenging. Table 4 shows that when these three controllers are compared, the error values are zero. However, the settling times for all controllers are 8 s. However, the maximum overshoot value is the lowest for the 2DOFPD, at 0.065. After fine-tuning, the controllers have zeroed their errors.

Table 4. Controller response

Controller	M_P	t_s (s)	e_{ss}
PD	0.25	8	0
2DOFPD	0.065	8	0
KDPD	0.212	8	0

Conclusion and Recommendations

In the quest to control a system characterized by time delays, this study used three distinct types of controllers and conducted a thorough comparative analysis of their performances (Özyetkin et al., 2018). Initially, parameters previously identified for a system with a KDPD controller in the literature were employed, yielding initial results. This was followed by trials using the same parameters but with a classical PID controller and a 2DOFPD controller, leading to a comparative assessment of the outcomes. In this phase, the KDPD controller emerged as the most effective, a conclusion that became evident upon a detailed examination of both error indices and errors of the controllers themselves.

Further explorations, involving the use of the fine-tuning function within Matlab/Simulink, enable automatic adjustments to the system. The controller parameters derived from this process are presented in Table 3. When the system was re-evaluated with these adjusted parameters, it was observed that, although there were no significant discrepancies in error indices among the three controllers, a more granular analysis of controller errors revealed that the 2DOFPD controller demonstrated a superior performance, particularly in terms of minimizing maximum overshoot. A comparison between Studies 1 and 2 revealed a notable reduction in error values and a decline in error index values. Moreover, there was a discernible decrease in the settling times. Significantly, the settling time, which was indeterminable in Study 1, was successfully calculated in Study 2.

The decision regarding the optimal type of controller for regulating systems is a critical aspect of control engineering. This study elucidates this point vividly. While the KDPD controller was found to be most effective in Study 1, the 2DOFPD controller yielded the best results in Study 2. This finding underscores the premise that determining the most effective controller without practical testing is not feasible.

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